Emergent Wavefunction Collapse from Compact Extra Dimensions

Timothy D. Andersen[1, *]

[1] *Georgia Institute of Technology, Atlanta, Georgia, 30332, USA*

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Abstract

This work investigates whether collapse-like dynamics can emerge from a fundamentally unitary quantum field theory defined on a compact extra dimension. A real scalar field on ($\mathbb{R}^{3,1} \times S^1_R$) is decomposed into Kaluza–Klein (KK) modes, with the zero mode treated as the observable system and the higher modes forming an inaccessible environment. Integrating out the KK tower yields a nonlocal influence functional in which the system couples to a composite bath operator. The resulting reduced dynamics is governed by multiplicative noise and memory kernels derived from correlators of the KK sector. The composite spectral density is gapped, with a correlation time set by the compactification scale ($\tau_B \sim R$). In the regime of weak coupling and small compactification radius, the dynamics admit a Markovian infrared limit within a stationary reference frame. In this limit, the reduced density matrix obeys a local master equation of Lindblad form with an operator proportional to (ϕ^2). For a vacuum KK sector, decoherence is transient and saturates due to the spectral gap. However, for a stationary non-vacuum KK sector with finite low-frequency occupation, the decoherence becomes approximately linear in time and quantitatively reproduces the visibility behavior of Continuous Spontaneous Localization (CSL). The full higher-dimensional theory remains unitary, and apparent irreversibility arises from tracing over the KK modes, spectral densification, and coarse-graining over bath correlations. This framework provides a first-principles mechanism by which collapse-like behavior can emerge as an effective open-system phenomenon without modifying the underlying quantum dynamics.

I. INTRODUCTION

The measurement problem remains one of the central conceptual challenges in quantum mechanics. While the Schrödinger equation predicts the unitary evolution of superpositions, measurements appear to select definite outcomes with probabilities given by the Born rule. Standard formulations of quantum theory do not provide a dynamical mechanism for this apparent collapse, leading to a variety of interpretational and phenomenological proposals.

Among these, spontaneous collapse models such as the Ghirardi–Rimini–Weber (GRW) model and its continuous extension, Continuous Spontaneous Localization (CSL), introduce

* andert@gatech.edu

stochastic modifications to the Schrödinger equation that dynamically suppress macroscopic superpositions [1][2]. These models successfully provide a unified description of microscopic and macroscopic behavior, but they do so by explicitly modifying quantum dynamics. In particular, they introduce irreversibility and stochasticity at a fundamental level, thereby breaking unitarity and time-reversal symmetry. While such modifications are phenomenologically viable, they lack a widely accepted microscopic origin.

A key open question is therefore whether collapse-like behavior can arise without modifying the underlying unitary dynamics of quantum mechanics. More broadly, one may ask whether the apparent irreversibility associated with measurement can emerge as an effective description of a larger, fundamentally reversible system.

In this work, we investigate this possibility in the context of quantum field theory with a compact extra dimension. Specifically, we consider a real scalar field on $(\mathbb{R}^{3,1} \times S^1_R)$, where the compactification leads to a discrete tower of Kaluza–Klein (KK) modes. The zero mode is treated as the observable sector, while the higher KK modes form an inaccessible environment. The interaction between these sectors arises from the quartic self-interaction of the higher-dimensional field, which induces a coupling between the zero mode and a composite operator constructed from the KK tower.

The resulting system is naturally described as an open quantum system. Using the Feynman–Vernon influence functional formalism [3] in a Keldysh contour framework [4], we integrate out the KK modes to obtain an effective description of the zero mode. Because the coupling is quadratic in both the system and bath fields, the reduced dynamics involve multiplicative noise and nonlocal memory kernels determined by correlators of a composite bath operator. This distinguishes the present framework from standard oscillator-bath models such as Caldeira–Leggett theory [5, 6].

A central feature of the construction is that the composite bath spectral density is gapped, with support beginning at twice the KK mass scale. As a result, a vacuum KK sector produces only transient decoherence that saturates at late times. Persistent collapse-like behavior requires a stationary non-vacuum occupation of the KK modes, characterized by finite low-frequency noise power. Importantly, this does not require the bath to satisfy a Kubo–Martin–Schwinger (KMS) [7, 8] condition or to be in thermal equilibrium; stationarity and spectral support are sufficient.

In the regime of weak coupling, small compactification radius, and dense KK spectrum, the bath correlation time becomes short compared to the system evolution timescale. Under these conditions, the nonlocal influence functional admits a Markovian infrared limit. The reduced density matrix then obeys a local master equation of Lindblad [9] form with an operator proportional to (ϕ^2). In the nonrelativistic limit, this operator reduces to a mass-density coupling, allowing direct comparison with CSL.

We show that, for a stationary non-vacuum KK sector, the resulting decoherence functional [10] produces an approximately linear decay of coherence and a visibility function that closely matches CSL predictions over experimentally relevant timescales. At the same time, the full higher-dimensional theory remains unitary. Apparent irreversibility arises from tracing over inaccessible KK modes, spectral densification as the compactification scale decreases, and coarse-graining over rapidly oscillating bath correlations. Recoherence is present in the exact dynamics due to the discrete spectrum but is strongly suppressed in the dense-spectrum limit.

This framework therefore provides a concrete example in which collapse-like dynamics emerge as an effective open-system phenomenon within a fundamentally unitary quantum field theory. It suggests that modifications of quantum mechanics may not be necessary to account for the emergence of classicality, but instead that such behavior can arise from hidden degrees of freedom and appropriate coarse-graining limits.

The paper is organized as follows. In Sec. II, we derive the effective dynamics of the zero mode using the influence functional formalism and compute the associated noise and dissipation kernels. Section III analyzes the role of recoherence and the emergence of effective irreversibility. Section IV discusses the regime of validity of the approximations, renormalization, Lorentz covariance, and experimental constraints, including comparisons to CSL. Section V concludes with a discussion of implications and directions for future work.

II. QUANTUM STOCHASTIC ϕ^4 THEORY

Consider the real scalar field $\Phi(x, \sigma)$ on $\mathbb{R}^{3,1} \times S^1_R$ with action,

$$S[\Phi] = \int d^4x \int_0^{2\pi R} d\sigma \left[\frac{1}{2} \partial_M \Phi \partial^M \Phi - \frac{1}{2} m_5^2 \Phi^2 - \frac{\lambda_5}{4!} \Phi^4 \right] \tag{1}$$

where $M = (\mu, 5)$, $\mu = 0, 1, 2, 3$ and $\sigma \sim \sigma + 2\pi R$.

Because the fifth dimension is compact, the field admits a KK expansion,

$$\Phi(x,\sigma) = \frac{1}{\sqrt{2\pi R}} \sum_{n\in\mathbb{Z}} \phi_n(x) e^{in\sigma/R} \tag{2}$$

for a real 5D field. The free part of the action becomes,

$$S_0 = \sum_n \int d^4x \left[\frac{1}{2}\partial_\mu \phi_n \partial^\mu \phi_{-n} - \frac{1}{2} m_n^2 \phi_n \phi_{-n} \right] \tag{3}$$

with KK masses,

$$m_n^2 = m_5^2 + \frac{n^2}{R^2}. \tag{4}$$

The quartic interaction reduces to

$$S_{\text{int}} = -\frac{\lambda}{4!} \int d^4x \sum_{n_1+n_2+n_3+n_4=0} \phi_{n_1}\phi_{n_2}\phi_{n_3}\phi_{n_4} \tag{5}$$

and

$$\lambda = \frac{\lambda_5}{2\pi R}.$$

Because the model involves a system of interest and an environment where one desires to reduce the equations for the system-cum-environment to one in which only the system appears, an open quantum system formalism is appropriate. For simple systems with bilinear interactions and additive noise, the Caldeira-Leggett [5][6] may be used, but for more complex, non-linear interactions and multiplicative noise such as this one, other approaches are needed.

A. Feynman-Vernon Langevin Equation

Separate the observable sector $n = 0$ mode from the hidden KK sector by writing,

$$\phi(x) \equiv \phi_0(x), \quad \chi_n(x) \equiv \phi_n(x),\, n \neq 0 \tag{6}$$

Then, the action is the sum of the three actions, (1) the system $n = 0$ mode, (2) the internal "bath" action, and (3) the coupling between them,

$$S = S_{\text{sys}}[\phi] + S_{\text{bath}}[\chi] + S_{\text{coup}}[\phi, \chi].$$

The system action is the standard 4D ϕ^4 theory action,

$$S_{\text{sys}}[\phi] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m_0^2 \phi^2 - \frac{\lambda}{4!}\phi^4 \right]. \tag{7}$$

To the lowest non-trivial order, the coupling between the zero mode and the KK tower contains,

$$S_{\rm coup}[\phi,\chi] = -\frac{\lambda}{4}\int d^4x\,\phi^2(x)\sum_{n\neq 0}\chi_n(x)\chi_{-n}(x) + \cdots \tag{8}$$

where the omitted terms are cubic and quartic in the bath fields and have higher order weak coupling.

The key point is that the interaction is quadratic in the system field and the bath fields. Therefore, the correct framework is the Feynman-Vernon influence functional for a composite bath operator [3],

$$\mathcal{B}(x) \equiv \sum_{n\neq 0}\chi_n(x)\chi_{-n}(x) \tag{9}$$

with interaction,

$$S_{\rm coup} = -\int d^4x\, J[\phi(x)]\mathcal{B}(x), \quad J[\phi] = \frac{\lambda}{4}\phi^2. \tag{10}$$

This form is sufficient to derive the effective dynamics of the zero mode using Keldysh contour theory, which is described in more detail in Appendix A.

Let $\phi^{\pm}$ and $\chi_n^{\pm}$ be the split degrees of freedom. The generating functional based on double-path integrals is [3],

$$Z = \int D\phi^{\pm}\,\exp[i(S_{\rm sys}[\phi^+] - S_{\rm sys}[\phi^-])]\mathcal{F}[\phi_+,\phi_-].$$

where the influence functional is,

$$\begin{aligned}\mathcal{F}[\phi_+,\phi_-] \;=\; & \int D\chi_+ D\chi_- \\ & \exp\Big[i(S_{\rm bath}[\chi_+] + S_{\rm coup}[\phi_+,\chi_+] - \\ & S_{\rm bath}[\chi_-] - S_{\rm coup}[\phi_-,\chi_-]\Big]\rho_B\end{aligned} \tag{11}$$

For a sufficiently large number of dense KK modes, which enable the central limit theorem, and weak coupling, the leading dynamics are captured by 2-point functions. (This approximation is assumed for the initial reduced KK state since interactions between modes generate higher cumulants than retained here.) This allows one to assume the KK bath is initially Gaussian and stationary. An expansion to second order in λ gives,

$$\begin{aligned}\log\mathcal{F} \;=\; & -\frac{i\lambda}{4}\int d^4x\,(\phi_+^2(x) - \\ & \phi_-^2(x))\langle\mathcal{B}(x)\rangle - \frac{\lambda^2}{32}\int d^4x d^4x'\mathcal{K}(x,x')\end{aligned} \tag{12}$$

where

$$\mathcal{K}(x,x') = \sum_{a,b=\pm} ab\phi_a^2(x)\phi_b^2(x')\langle T_C \mathcal{B}_a \mathcal{B}_b(x')\rangle_c \tag{13}$$

where T_C denotes the contour ordering and the subscript c is the connected correlator.

The first term renormalizes the mass and local couplings of the zero mode. Define,

$$\delta m^2 = \frac{\lambda}{2}\langle \mathcal{B}(x)\rangle \tag{14}$$

so that the contribution may be absorbed into a renormalized system action.

Apply the Keldysh rotation [4],

$$J_c = \frac{\lambda}{4}(\phi_+^2 + \phi_-^2), \quad J_\Delta = \frac{\lambda}{4}(\phi_+^2 - \phi_-^2).$$

The Influence Functional (IF) action [3] is,

$$\begin{aligned} S_{\text{IF}} &= \int d^4x d^4x' J_\Delta(x)[D_R(x-x')J_c(x') \\ &+ i\int d^4x d^4x' J_\Delta(x) N_B(x-x') J_\Delta(x') \end{aligned} \tag{15}$$

The retarded and noise kernels (anti-symmetric and symmetric) are [4],

$$D_R(x-x') = \Theta(t-t')\langle[\mathcal{B}(x),\mathcal{B}(x')]\rangle \tag{16}$$

$$N_B(x-x') = \frac{1}{2}\langle\{\delta\mathcal{B}(x),\delta\mathcal{B}(x')\}\rangle. \tag{17}$$

where $\delta\mathcal{B}(x) = \mathcal{B}(x) - \langle\mathcal{B}(x)\rangle$ is the fluctuation.

Now, the bath correlators need to be evaluated. At leading order, the KK modes are free. Therefore, Wick's theorem applies. Since,

$$\mathcal{B}(x) = \sum_{n\neq 0}\chi_n(x)\chi_{-n}(x) \tag{18}$$

the connected symmetrized correlator is,

$$\langle\delta\mathcal{B}(x)\delta\mathcal{B}(x')\rangle = 2\sum_{n\neq 0} G_n^>(x-x')G_n^<(x-x'). \tag{19}$$

and the commutator is determined by the antisymmetric part. Therefore,

$$\begin{aligned} N_B(x-x') &= \frac{1}{2}\sum_{n\neq 0}[G_n^>(x-x')G_n^<(x-x') + \\ &\quad G_n^<(x-x')G_n^>(x-x')] \end{aligned} \tag{20}$$

For a stationary bath (which has been assumed for a particular choice of reference frame), the Fourier transform of the noise kernel is convenient,

$$N_B(\omega, \mathbf{q}) = \frac{1}{2} \sum_{n\neq 0} \int \frac{d^4p}{(2\pi)^4} G_{K,n}(p) G_{K,n}(q-p) \tag{21}$$

where G_K is the Keldysh propagator. The retarded kernel 16 may also be computed from a retarded and a Keldysh propagator,

$$D_R(\omega, \mathbf{q}) = \sum_{n\neq 0} \int \frac{d^4p}{(2\pi)^4} G_{R,n}(p) G_{K,n}(q-p) \tag{22}$$

In a vacuum or stationary state, the support of these kernels begins at the two-particle threshold,

$$\omega^2 - \mathbf{q}^2 \geq 4m_n^2.$$

This threshold is important because the bath operator $\mathcal{B} \sim \chi_n \chi_{-n}$ creates or annihilates pairs of KK quanta. Therefore, the effective spectral density is not that of a linearly coupled oscillator bath as in the Caldeira-Leggett theory [6]. Instead, it is a composite-bath spectral density with a two-particle continuum.

The spectral function can be written as ,

$$\begin{aligned} \rho_B(\omega, \mathbf{q}) &\equiv \int d^4x\, e^{i(\omega t - \mathbf{q}\cdot\mathbf{x})} \langle [\mathcal{B}(x), \mathcal{B}(0)] \rangle \\ &= \sum_{n\neq 0} \rho_n(\omega) \end{aligned} \tag{23}$$

For a real 5D scalar field, $\chi_{-n} = \chi_n^\dagger$. Rewrite the nonzero KK sector as two real fields for each $n > 0$ such that $\chi_n, \chi_{-n} \to \chi_{n,1}\chi_{n,2}$. Then the bath operator is,

$$\mathcal{B}(x) = \sum_{n>0} \sum_{a=1}^{2} \chi_{n,a}^2(x).$$

For a single real scalar field χ of mass m_n, the operator χ^2 creates two identical quanta. The positive frequency spectral density is,

$$\rho^+_{\chi^2}(\omega, \mathbf{q}) = 2 \int \frac{d^3p}{(2\pi)^3 2E_{\mathbf{p}}} \frac{d^3p'}{(2\pi)^3 2E_{\mathbf{p}'}} (2\pi)^4 \delta(q - p - p')$$

where $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m_n^2}$ and the delta function is over $p^\mu = (\omega, \mathbf{p}), \quad p'^\mu = (\omega, \mathbf{p}')$ with $p = \sqrt{p_\mu p^\mu}$. The factor of 2 comes from the matrix element of χ^2 between the vacuum and two-particle state, including the identical-particle symmetry factor.

The standard two-body Lorentz invariant phase space integration [11] gives,

$$\rho^{+}_{\chi^2}(\omega, \mathbf{q}) = \frac{1}{4\pi}\sqrt{1 - \frac{4m_n^2}{s}}\Theta(s - 4m_n^2)$$

with $s = \omega^2 - q^2$.

Since each positive KK level $n > 0$ supplies two real fields, the contribution per KK level is double that,

$$\rho^{+}_{B,n}(\omega, \mathbf{q}) = \frac{1}{2\pi}\sqrt{1 - \frac{4m_n^2}{s}}\Theta(s - 4m_n^2).$$

Restoring the oddness of the commutator spectral function,

$$\rho_B(\omega, \mathbf{q}) = \mathrm{sgn}(\omega)\sum_{n>0}\frac{1}{2\pi}\sqrt{1 - \frac{4m_n^2}{s}}\Theta(s - 4m_n^2).$$

At zero spatial momentum $\mathbf{q} = \mathbf{0}$,

$$\rho_B(\omega) = \mathrm{sgn}(\omega)\sum_{n>0}\frac{1}{2\pi}\sqrt{1 - \frac{4m_n^2}{\omega^2}}\Theta(|\omega| - 2m_n)$$

In a continuum approximation of dense modes, this leads to an evaluation of the spectral density,

$$\rho_B(\omega) = R\omega/16, \tag{24}$$

which will be useful for later, but the discrete spectrum implies that the KK sector is gapped,

$$\rho_B(\omega) = 0 \quad \text{for} \quad |\omega| < 2m_1 \tag{25}$$

with a mass gap of $m_1 = \sqrt{m_5^2 + 1/R^2}$, the lightest KK particle mass.

This means the dissipative kernel has the generic form,

$$D_R(\omega) = \int \frac{d\omega'}{2\pi}\frac{\rho_B(\omega')}{\omega - \omega' + i0^+} \tag{26}$$

and the noise kernel satisfies,

$$N_B(\omega) = \frac{1}{2}F(\omega)\rho_B(\omega) \tag{27}$$

only if the bath is approximately stationary. No KMS condition, however, is assumed; therefore, the form of $F(\omega)$ is not known. In a strict, vacuum state, $F(\omega) = 1$ in which case,

$$N_{\text{vac}}(\omega) = \frac{1}{2}\rho_B(\omega). \tag{28}$$

The spatial noise kernel when $t = t'$ is given by,

$$N(\mathbf{r}) = \frac{1}{2}\langle\{\delta\mathcal{B}(\mathbf{r},0), \delta\mathcal{B}(0,0)\}\rangle \tag{29}$$

At leading order, Wick factorization gives the sum of squares of the Green's functions for $\nabla^2 + m_n^2$,

$$N_B(\mathbf{r}) = \frac{1}{2}\sum_{n\neq 0} G_n(\mathbf{r})^2. \tag{30}$$

where, in the free, unbounded case is,

$$G_n(\mathbf{r}) \sim \frac{e^{-m_n r}}{4\pi r}.$$

$$N_B(r) \sim \begin{cases} R/r^3 & r \ll R \\ e^{-2r/R}/r^2 & r \gg R. \end{cases} \tag{31}$$

Thus, there is a Yukawa-like correlation cutoff at long distances with inverse square law and a higher-dimensional, inverse cube fall off at short scales with no cutoff. This means that, within the compact dimension, noise correlations are long-range, but for larger scales, they are cut off by the compact dimension radius.

To characterize the noise influencing the observable sector, the imaginary part of the influence action 15 can be represented by a Hubbard-Stratonovich [12][13] field $\xi(x)$

$$\begin{aligned} &\int d^4x d^4x' \, J_\Delta(x) N_B(x-x') J_\Delta(x') \\ = &\int D\xi \, \exp\Big(-\frac{1}{2}\int d^4x d^4x' \, \xi(x) N_B^{-1}(x-x')\xi(x') \\ + \; &i\int d^4x \, \xi(x) J_\Delta(x)\Big). \end{aligned} \tag{32}$$

The last term is, in terms of ϕ,

$$i\int d^4x \, \xi(x) J_\Delta(x) = i\frac{\lambda}{2}\int d^4x \, \xi(x)\phi_c(x)\phi_\Delta(x),$$

(up to order ϕ_Δ^3) where

$$\phi_\Delta = \frac{1}{2}(\phi_+ - \phi_-), \quad \phi_c = \frac{1}{2}(\phi_+ + \phi_-).$$

Varying with respect to ϕ_Δ and setting $\phi_\Delta = 0$ yields the semi-classical Langevin equation,

$$\boxed{\Box\phi(x) + m_R^2\phi(x) + \frac{\lambda_R}{3!}\phi^3(x) + \frac{\lambda^2}{8}\phi(x)\int d^4x' D_R(x-x')\,\phi^2(x') = \frac{\lambda}{2}\phi(x)\,\xi(x)} \tag{33}$$

where $\Box$ is the d'Alembertian and the noise correlation given by the noise kernel

$$\langle\xi(x)\xi(x')\rangle = N_B(x-x'). \tag{34}$$

and zero mean$\langle\xi(x)\rangle = 0$.

This Langevin equation (along with forms for the kernels) is the central result of the paper. The zero mode obeys a nonlocal, multiplicative-noise Langevin equation generated by integrating out the hidden KK tower. Collapse-like suppression of off-diagonal coherence, therefore, arises because of the reduced density matrix as it decoheres under the influence of the kernel N, even though the full 5D theory remains unitary. This admits a Markovian limit, but the form of the noise kernel plays an important role in determining whether collapse dynamics are sufficient to agree with experimental bounds.

B. Markovian Limit

The evolution is genuinely non-Markovian, but a Markovian limit exists. If the zero mode varies on timescales much longer than the bath memory time, the retarded kernel may be expanded around low frequency,

$$D_R(t-t', \mathbf{x}-\mathbf{x}') \approx \eta\delta(t-t')\delta^{(3)}(\mathbf{x}-\mathbf{x}') \tag{35}$$

as a leading local approximation. The noise kernel becomes

$$N(t-t', \mathbf{x}-\mathbf{x}') \approx 2D\delta(t-t')\delta^{(3)}(\mathbf{x}-\mathbf{x}') \tag{36}$$

In this case the stochastic equation reduces to

$$\Box\phi + m_R^2\phi + \frac{\lambda_R}{3!}\phi^3 + 2\eta\phi^3 = \phi\xi \qquad \langle\xi(x)\xi(x')\rangle = 2D\delta^{(4)}(x-x'). \tag{37}$$

This local form is an effective infrared approximation. The full theory has finite-memory correction controlled by the KK thresholds and the compactification scale R.

This occurs if the bath correlation time τ_B is much shorter than the system evolution time τ_S. The natural bath correlation time is set by the lightest accessible bath threshold as long as $m_5 R \ll 1$,

$$\tau_B \sim \frac{1}{2m_1} \sim R/2, \qquad m_5 R \ll 1$$

This is a reasonable assumption. For example, a compactification scale 1 TeV corresponds to $R \sim 2 \times 10^{-19}$ m although collider bounds on KK compactification scales are model-dependent [14].

The system timescale depends on the characteristic timescale $\tau_S \sim 1/\Omega_S$ with Ω_S the characteristic system frequency. The secular (rotating-wave) approximation, where rapidly oscillating terms in the interaction picture may be averaged out, is given by

$$\Omega_S \tau_B \ll 1 \implies \Omega_S \ll 2m_1 \sim \frac{2}{R} \tag{38}$$

which corresponds to retaining only energy-conserving contributions in the dissipator and ensures that the reduced dynamics take Lindblad form below.

In the local limit, the local diffusion function is,

$$\mathcal{D}(x) = \phi(x) \int d^4x'\, D_R(x - x')\phi^2(x'). \tag{39}$$

Write $x' = (t - \tau, \mathbf{x} - \mathbf{y})$ such that,

$$\mathcal{D}(x) = \phi(x) \int_0^\infty d\tau \int d^3dy\, D_R(\tau, \mathbf{y})\phi^2(t - \tau, \mathbf{x} - \mathbf{y}). \tag{40}$$

Expand ϕ^2 around x,

$$\phi^2(t - \tau, \mathbf{x} - \mathbf{y}) = \phi^2(x) - \tau\partial_t\phi^2(x) - y^i\partial_i\phi^2(x) + \frac{\tau^2}{2}\partial_t^2\phi^2(x) + \cdots$$

By rotational symmetry, the odd spatial term integrates to zero. Therefore, the local diffusion is,

$$\begin{aligned} \mathcal{D}(x) \;&=\; \phi(x)\Big[\mu_0\phi^2(x) - \mu_1\partial_t\phi^2(x) \\ &+\; \frac{\mu_2}{2}\partial_t^2\phi^2(x) + \nu_2\nabla^2\phi^2(x) + \cdots\Big] \end{aligned} \tag{41}$$

where

$$\mu_n = \int_0^\infty d\tau \int d^3y \tau^n D_R(\tau, \mathbf{y}).$$

The first term in 41 renormalizes the local potential. The second is a leading dissipative correction. Higher terms are suppressed by powers of the bath correlation time and length. Therefore, a local approximation with no memory is valid when $\mu_2\Omega_S^2 \ll \mu_1\Omega_S \ll \mu_0$ or $\tau_B\Omega_S \ll 1, \quad k_S L_B \ll 1$ where

$$\tau_B \sim L_B \sim \frac{1}{2m_1} \sim R/2. \tag{42}$$

and k_S is the characteristic spatial momentum of the system.

By the Born approximation, if $\lambda^2 \ll 1$, $\lambda^2|D_R|\tau_B \ll 1$ so that the bath state is only weakly perturbed by the zero mode, the Markov approximation is justified.

In a thermal regime where the bath has a temperature $T = 1/\beta$ (which is not assumed in this paper since the KMS condition has not been justified), the Fluctuation-Dissipation Theorem (FDT) is [15][4],

$$N_B(\omega, k) = \frac{1}{2}\coth\left(\frac{\beta\omega}{2}\right)\mathrm{Im}D_R(\omega, k) \tag{43}$$

or equivalently

$$N_B(\omega, \mathbf{q}) = \frac{1}{2}\coth\left(\frac{\beta\omega}{2}\right)\rho_B(\omega, \mathbf{q}). \tag{44}$$

At zero temperature $\coth(\beta\omega/2) \to \mathrm{sgn}(\omega)$.

C. Master and Lindblad Equations

Given the Born-Markov approximation, a local master equation may be derived. Under the secular approximation, this may have a Lindblad form. Consider that the exact reduced dynamics are non-Markovian and non-local,

$$\dot{\rho} = \int_0^t ds\, \mathcal{K}(t-s)\rho(s)$$

In an interaction picture with an interaction Hamiltonian,

$$H_{\text{int}} = J \otimes \mathcal{B}, \qquad J = \frac{\lambda}{4}\phi^2,$$

the second order Born-Markov master equation [15] is a Redfield-type equation [16] with non-Markovian structure

$$\begin{aligned}\frac{d\rho}{dt} &= -i[H_{\text{eff}}, \rho] - \\ &\int_0^\infty ds\, \big(C(s)[J, [J(-s), \rho]] - \\ &i\chi(s)[J, \{J(-s), \rho\}]\big) \end{aligned} \tag{45}$$

where the bath time symmetric, noise and antisymmetric, dissipation correlators are,

$$C(s) = \frac{1}{2}\langle\{\delta\mathcal{B}(s), \delta\mathcal{B}(0)\}\rangle,\ \chi(s) = \frac{i}{2}\langle[\mathcal{B}(s), \mathcal{B}(0)]\rangle. \tag{46}$$

This is not yet able to lead to Lindblad form.

Non-local terms in 45 involve phase factors $e^{i(\omega-\omega')s}$. Since the secular approximation 38 is valid, these fast oscillatory terms average out to zero if $\omega \neq \omega'$. Therefore, because of the Born, Markov, and secular approximations, the master equation becomes Markovian,

$$\frac{d\rho}{dt} = -i[H_{\text{eff}}, \rho] - \gamma[J, [J, \rho]], \tag{47}$$

with $\gamma = \int_0^\infty ds\, C(s)$ being the time-cumulative noise correlator.

The integral of the bath correlator $C(s)$ converges because of exponential suppression above the KK scale, rendering $\gamma < \infty$.

The master equation can be written in Lindblad form,

$$\frac{d\rho}{dt} = -i[H_{\text{eff}}, \rho] + L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}, \tag{48}$$

with $L = \sqrt{2\Gamma_\phi}\phi^2$ and $\Gamma_\phi = \frac{\lambda^2\gamma}{16}$.

Note that the Lindblad is in the magnitude of the field ϕ^2, not the signed amplitude ϕ, which is based on energy-density, not position-localization. The connection to CSL is made via the nonrelativistic limit in Section IV F.

D. Decoherence and Visibility

At zero temperature, the KK vacuum induces nonlocal vacuum decoherence and renormalization. A local irreversible Markov limit exists when the zero mode interacts with the bath on timescales long compared to the timescale implied by the first bath mode, and in a regime where the coarse-grained composite spectrum is effectively continuous. This makes the KK vacuum a structured, gapped (25) composite environment whose memory time is set by the compactification radius.

The decoherence exponent is,

$$\Gamma[\phi_+, \phi_-] = \frac{1}{2} \int d^4x d^4x' \, J_\Delta(x) N_B(x - x') J_\Delta(x'). \tag{49}$$

Therefore, substituting $J = \frac{\lambda}{4}\phi^2$ into J_Δ,

$$\begin{aligned} \Gamma[\phi_+, \phi_-] \; &= \; \frac{\lambda^2}{32} \int d^4x d^4x' \, (\phi_+^2(x) - \phi_-^2(x)) \\ & \qquad N_B(x - x')(\phi_+^2(x') - \phi_-^2(x')). \end{aligned} \tag{50}$$

In the Markovian approximation and under slow system relaxation relative to bath memory $\Gamma\tau_B \ll 1$, the symmetric bath correlator becomes local,

$$N_B(x - x') \approx 2D\delta^{(4)}(x - x') \tag{51}$$

in which case the decoherence is,

$$\Gamma \approx \frac{\lambda^2 D}{16} \int d^4x \, (\phi_+^2(x) - \phi_-^2(x))^2. \tag{52}$$

Therefore, the combined justification for a CSL-like effective theory is that small R gives $\tau_B \sim R \ll 1$, small λ gives $\Gamma \propto \lambda^2$ also small, $\Gamma\tau_B \ll 1$ which justifies a memory-less Markov approximation.

The vacuum state decoherence function, however, is transient with rapid decoherence followed by saturation, meaning that decoherence and collapse are halted because of the gap. Entanglement mixing simply cannot escape sufficiently into a vacuum.

To see this, define the visibility function as,

$$\mathcal{V}(t) = e^{-\Gamma(t)}$$

for some reasonable choice of conditions and Figure 1 shows how decoherence saturates while CSL predicts a linear decrease.

E. Stationary Gaussian Mixed State KK sector

A strict zero-temperature KK vacuum produces only transient decoherence because the composite bath spectrum is gapped (as in 25). While a dense KK mode continuum approximation eliminates this gap, the continuum model is not physically applicable when

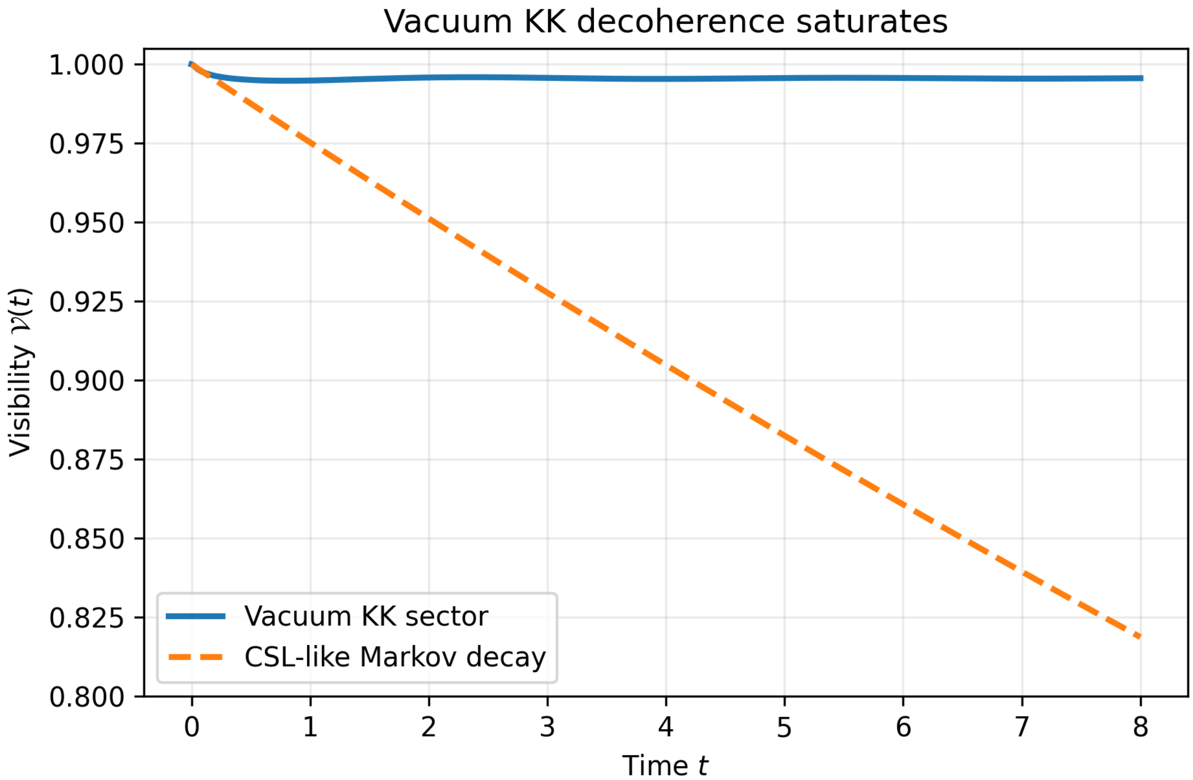

FIG. 1. Given vacuum KK density, decoherence is transient rather than persistent as with CSL. This is because the bath spectrum is gapped. Parameters: $R = 1$, $\Omega = 40$, $\alpha = 0.18$ (for equation 56 below), and $\gamma_{\rm CSL} = 0.025$. All quantities shown in natural units with $\hbar = c = 1$ and generated from vacuum noise 28

computing visibility for realistic collapse dynamics. This suggests that a vacuum KK sector is not consistent with observation. To develop persistent collapse-like dynamics, a more general case is considered now where the inaccessible KK sector is in a stationary Gaussian state with nonzero occupation numbers. This does not require assuming a thermodynamic bath. Rather, the KK sector could be populated by (1) compactification-era particle production, (2) coarse-graining over inaccessible geometric degrees of freedom, or (3) by initial entanglement between visible and hidden sectors.

Let the density matrix for the KK modes not be a zero-temperature vacuum state

$$\rho_{\rm KK} \neq |0\rangle\langle 0|.$$

Instead, let it be a stationary Gaussian mixed state with mode occupations,

$$\rho_{\text{KK}} = \prod_{n,\mathbf{k}} \rho_{n,\mathbf{k}}, \qquad \langle a^\dagger_{n,\mathbf{k}} a_{n,\mathbf{k}} \rangle = f_{n,\mathbf{k}}.$$

The Keldysh propagator then becomes,

$$G_K(\omega, \mathbf{k}) = (1 + 2f_{n,\mathbf{k}})\rho_n(\omega, \mathbf{k}). \tag{53}$$

This changes the noise kernel 21 to be

$$N_B(\omega, \mathbf{q}) = \frac{1}{2} \sum_{n \neq 0} d\Pi_{\mathbf{p}} d\Pi_{\mathbf{p}'} \mathcal{F}_n(p, p')(2\pi)^4 \delta^{(4)}(q - p - p')$$

with

$$\mathcal{F}_n(p, p') = 1 + f_{n,p} + f_{n,p'} + 2f_{n,p} f_{n,p'} \tag{54}$$

the occupation correlation,

$$d\Pi_{\mathbf{p}} = \int \frac{d^3p}{(2\pi)^3 2\omega_{n,\mathbf{p}}},$$

and

$$p^\mu = (\omega_{n,\mathbf{p}}, \mathbf{p}) \quad p'^\mu = (\omega_{n,\mathbf{p}'}, \mathbf{p}').$$

The vacuum contribution corresponds to $f = 0$. Persistent Markovian decoherence requires nonzero low-frequency noise power after coarse-graining. If the stationary occupation produces an approximately infrared-enhanced fluctuation ratio,

$$F(\omega, \mathbf{q}) = \frac{2N_B(\omega, \mathbf{q})}{\rho_B(\omega, \mathbf{q})},$$

then this is possible.

Only in the special case where,

$$F(\omega) = \coth\left(\frac{\omega}{2T_{\text{eff}}}\right)$$

may T_{eff} be interpreted as an effective temperature. Otherwise, F is simply a nonthermal occupation-weighted fluctuation ratio.

In the dense, KK continuum approximation,

$$\rho_B(\omega) \simeq \frac{R\omega}{16} e^{-\omega/\Omega}$$

where using 24 for the normalization, R is the compact radius, and Ω is the UV cutoff.

In the special case of a thermal-like stationary occupation where $\omega \ll \Theta_{\rm KK}$,

$$N_B(\omega) = \frac{1}{2}F(\omega)\rho_B(\omega) \simeq \frac{R\Theta_{\rm KK}}{16}, \tag{55}$$

where $\Theta_{\rm KK}$ is a phenomenological noise scale determined by the stationary distribution (which if this were a thermal bath would be the effective temperature).

Thus, the noise spectrum becomes approximately white in the infrared. The KK decoherence functional is then computed as,

$$\Gamma_{\rm KK}(t) = \alpha \int_0^\infty \frac{d\omega}{2\pi}\, N_B(\omega)\frac{2[1-\cos(\omega t)]}{\omega^2}. \tag{56}$$

where α is a pre-factor absorbing $\lambda^2/32$.

This gives a local Markovian decoherence functional

$$\Gamma_{12} = \frac{1}{2}\int d^4x d^4x'\, J_{\Delta,12}(x) N_B(x-x') J_{\Delta,12}(x') \tag{57}$$

where

$$J_{\Delta,12} = \frac{\lambda}{4}(\phi_1^2 - \phi_2^2). \tag{58}$$

In the local limit,

$$N_B(t-t') \simeq 2D\delta(t-t')$$

the decoherence is,

$$\Gamma_{12} \simeq \frac{\lambda^2 D}{16}\int_0^t dt' \int d^3x\, (\phi_1^2 - \phi_2^2)^2. \tag{59}$$

For stationary configurations $\partial_t\phi_1 = \partial_t\phi_2 = 0$ this becomes, $\Gamma_{12}(t) = \gamma_{\rm KK} t$, with

$$\gamma_{\rm KK} = \frac{\lambda^2 D}{16}\int d^3x\, (\phi_1^2 - \phi_2^2)^2. \tag{60}$$

The visibility is then,

$$\mathcal{V}(t) = e^{-\Gamma_{12}(t)} \tag{61}$$

which has the same form as the CSL visibility law, $\mathcal{V}_{\rm CSL}(t) = e^{-\gamma_{\rm CSL} t}$ where one identifies the two pre-factors $\gamma_{\rm CSL} = \gamma_{\rm KK}$.

Assuming a spectral density of

$$\rho_B(\omega) = \frac{R\omega}{16} e^{-\omega/\Omega}\Theta(\omega - \omega_{\rm IR})$$

a fluctuation ratio of

$$F(\omega) \approx \frac{2\Theta_{\rm KK}}{\omega}, \tag{62}$$

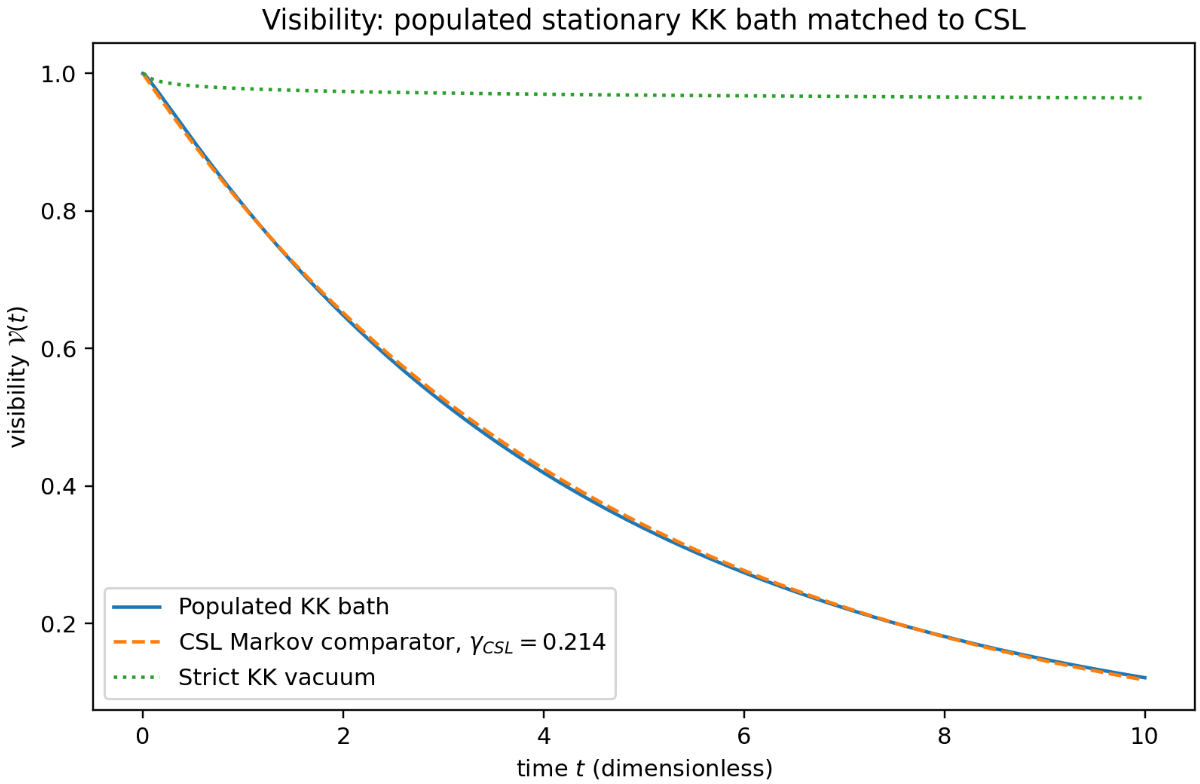

FIG. 2. The populated KK sector produces a quantitative agreement to CSL visibility while the vacuum solution saturates. This confirms the need for a populated KK sector in order to produce collapse-like dynamics like CSL. Parameters: $R = 1$, $\Omega = 40$, $\omega_{\rm IR} = 0.02$, $\Theta_{\rm KK} = 192$, $\alpha = 0.038$ and $\gamma_{\rm CSL} = 0.214$.

a noise spectrum $N_B(\omega) = \frac{1}{2}F(\omega)\rho_B(\omega)$ a KK decoherence functional 56, a strict vacuum noise of $N_{\rm vac}(\omega) = \frac{1}{2}\rho_B(\omega)$, then the collapse rate is,

$$\gamma_{\rm KK} = \frac{\lambda^2 R\Theta_{\rm KK}}{256}\mathcal{I}_{12} \tag{63}$$

where

$$\mathcal{I}_{12} = \int d^3x\,(\phi_1^2 - \phi_2^2)^2. \tag{64}$$

Given these assumptions, quantitative agreement to CSL over the plotted time interval may be achieved as long as parameters are carefully chosen, as in Figure 2 for the populated KK sector solution versus the vacuum. These parameters are for illustration only; mapping to physical units is discussed in Section IV.

III. UNITARY EVOLUTION, RECOHERENCE, AND IRREVERSIBILITY

The full theory is defined by the 5D Hamiltonian,

$$H = H_{\rm sys} + H_{\rm KK} + H_{\rm int} \tag{65}$$

with

$$H_{\rm int} = \int d^3x\ J(x)\mathcal{B}(x). \tag{66}$$

The total density matrix evolves unitarily,

$$\rho_{\rm tot}(t) = \mathcal{U}(t)\rho_{\rm tot}(0)\mathcal{U}^\dagger(t), \qquad \mathcal{U}(t) = e^{-iHt}.$$

Therefore, information is never destroyed, and any apparent collapse must arise from tracing out KK modes. This establishes that the zero-mode sector is an effective behavior.

The observable state is the reduced density matrix,

$$\rho(t) = {\rm Tr}_{\rm KK}\rho_{\rm tot}(t). \tag{67}$$

From the influence functional 15, the off-diagonal suppression is governed by 49. The key feature is that the noise kernel has finite temporal support,

$$N(t) \sim \sum_{n\neq 0} e^{-2im_n t}F_n(t). \tag{68}$$

Because the KK spectrum is discrete (or quasi-discrete), the kernel is not strictly decaying but exhibits oscillatory behavior.

The consequence of this is that the reduced density matrix is not strictly monotonic, $\rho_{12}(t) \sim e^{-\Gamma(t)}$ where $\Gamma(t)$ is given by 56. Instead, it contains oscillatory contributions.

This means that the decoherence is not guaranteed to increase, $\frac{d\Gamma}{dt} \not\geq 0$, in general, which is analogous to fluctuation theorems in systems that stray from equilibrium. As in those systems, decreasing entropy occurs over short timescales, so here, recoherence is generically present in the exact reduced dynamics.

Recoherence reflects the fact that information is stored in a finite, structured environment and phase correlations can return to the system. This is analogous to Poincaré recurrences [17], finite-size quantum baths, and spin-boson models with discrete spectra.

In this model, the recoherence comes from $\omega \sim 2m_n$ threshold oscillations in the composite KK bath.

The dominant time scale is set by the level spacing,

$$\tau_{\text{rec}} \sim \frac{1}{\Delta m} \sim R.$$

Given that R is small, this leads to rapid oscillations and short memory without visible recoherence.

Recoherence becomes unobservable for three reasons: (1) dense KK spectrum,

$$\sum_n e^{-2im_n t} \to \int dm e^{-2imt} \sim \delta(t)$$

after coarse-graining shortens recoherence timescales. (2) If the KK sector has occupation (as required to match CSL) where $f_{n,\mathbf{k}} \neq 0$, the phase correlations between modes are randomized, which makes recoherence even less likely. (3) Physical measurements that probe timescales such that $\Delta t \gg R/c$ will average out the recoherence oscillations.

Under the conditions that $\Omega_S R \ll 1$ and $\Gamma R \ll 1$, the noise kernel admits the Markov approximation with no memory. This eliminates recoherence, and the visibility is strictly monotonic. Irreversibility emerges from (1) coarse-graining, (2) dense KK spectrum, and (3) weak coupling.

Therefore, while the exact theory is unitary and the reduced non-Markovian theory allows decoherence and recoherence with no true collapse, under the Markov limit, decoherence is irreversible, and CSL-like behavior emerges. Thus, collapse is an infrared effective description of a unitary higher-dimensional theory.

A numerical simulation of visibility for different compactification scales shows the recoherence oscillations for decreasing values of R in Figure 3. For a large compactification radius, the KK spectrum is sparse, and the reduced dynamics exhibit strong oscillatory recoherence. The visibility periodically returns to unity because of the finite number of degrees of freedom in the interaction.

As the KK spectrum becomes dense, the phase correlations rapidly become dephased. The memory kernel becomes sharply peaked in time, suppressing recoherence.

Once recoherence is suppressed from dense KK spectrum and stationary occupation, the reduced dynamics produce the CSL-like behavior as in Figure 2, which is experimentally indistinguishable from CSL over relevant timescales.

To summarize: Collapse-like behavior does not arise from modifying quantum mechanics, but from a hierarchy of limits:

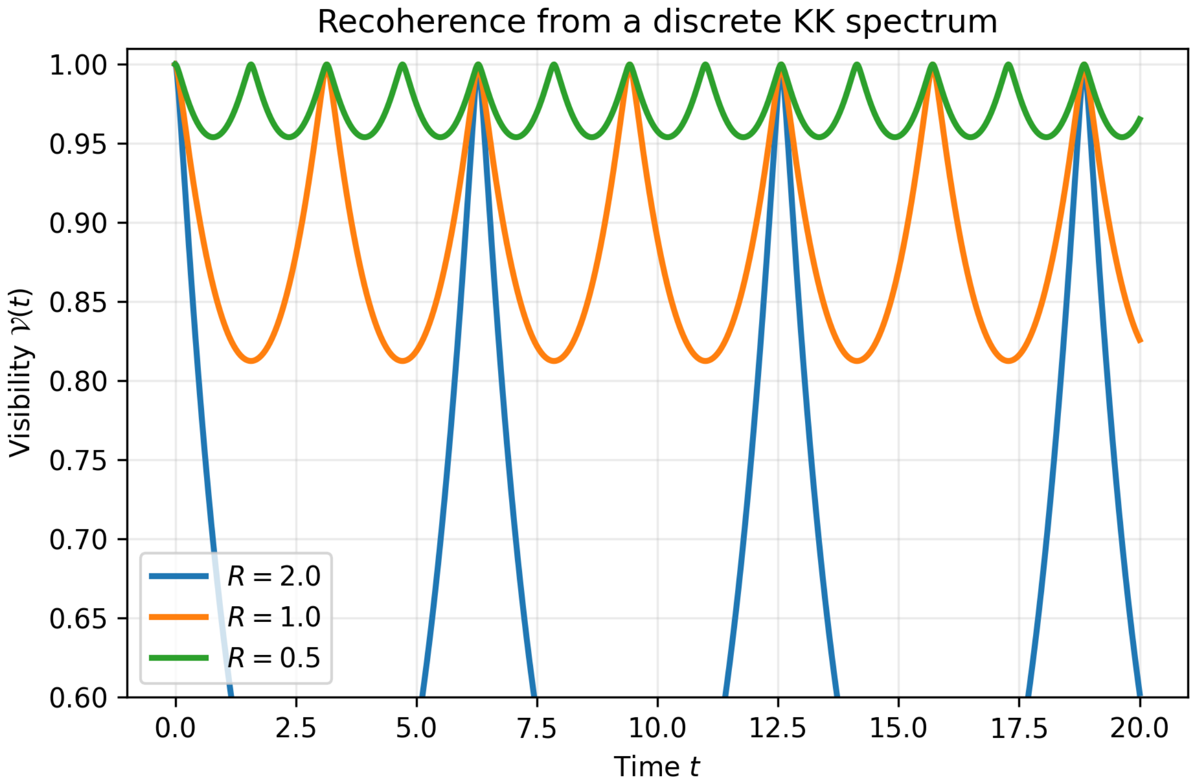

FIG. 3. Visibility oscillates widely for large R, indicating period returns to coherence, but these oscillators decrease with R showing how recoherence is suppressed as R becomes small using integral equation . Units are natural $\hbar = c = 1$.

- tracing over inaccessible KK modes,

- spectral densification as $R \to 0$,

- coarse-graining over rapid bath correlations, and

- stationary occupation of hidden degrees of freedom.

In this regime, the non-Markovian unitary dynamics reduce to an effectively irreversible Markovian master equation.

IV. VALIDITY, RENORMALIZATION, LORENTZ COVARIANCE, AND EXPERIMENTAL CONSTRAINTS

The derivation of the reduced dynamics relies on a controlled hierarchy of approximations. Given $\tau_B \sim (2m_1)^{-1} \sim R/2$ and $m_1^2 = m_5^2 + 1/R^2$ with $m_5 R \ll 1$ the following conditions define the regime where the results are quantitatively reliable:

1. Weak coupling (Born approximation): $\lambda^2 |D_R| \tau_B \ll 1, \quad \Gamma\tau_B \ll 1$.

2. Markov (short-memory) condition: $\Omega_S \tau_B \ll 1 \implies \Omega_S R \ll 1$.

3. Stationary non-vacuum KK sector: $f_{n,\mathbf{k}} \neq 0$ such that the symmetrized noise spectrum has nonzero low-frequency weight after coarse graining.

4. Infrared coarse-graining: observables are probed on timescales $\Delta t \gg \tau_B$ suppressing non-Markovian oscillations.

The assumption of Gaussian bath can be derived from (1) the large number of dense KK modes leading to central limit behavior, (2) weak coupling prevents distortion of the bath by the system, and (3) higher connected correlators scale like $1/N$. This approximation is assumed at the level of the initial reduced KK state; interactions among KK modes generate higher cumulants beyond the order retained here.

Under these conditions, non-local kernels admit derivative expansion and reduced dynamics becomes local in time, yielding the Markovian master equation,

$$\frac{d\rho}{dt} = -i[H_{\text{eff}}, \rho] - \Gamma_\phi[\phi^2, [\phi^2, \rho]]. \tag{69}$$

where Γ_ϕ is given by 70 below.

A. Dimensional Analysis

Natural units, $\hbar = c = 1$ has been used throughout except where included for emphasis. A dimensional analysis follows: the scalar field in 4+1 dimensions has mass dimension $[\Phi] = 3/2$, but after KK reduction the 4D modes, χ_n and ϕ have dimension 1. Therefore, the 5D quartic coupling has dimension $[\lambda_5] = \text{mass}^{-1}$. After compactification $\lambda = \lambda_5/(2\pi R)$. Therefore, $[\lambda] = \text{mass}^0$.

The composite bath operator $\mathcal{B} = \sum_{n\neq 0} \chi_n \chi_{-n}$ has dimension $\mathcal{B} = \text{mass}^2$. Therefore, the coupling $J\mathcal{B}$ where $J = \frac{\lambda}{4}\phi^2$ is dimension 4 as required.

The continuum KK spectral density (per unit volume) takes the form

$$\rho_B(\omega) \simeq \frac{R\omega}{16} e^{-\omega/\Omega}.$$

and is dimensionless. The noise scale Θ_{KK} from 62 has dimensions of mass.

The Markov noise strength is $D \simeq \frac{R\Theta_{\text{KK}}}{16}$ is dimensionless and the decoherence rate,

$$\Gamma_\phi \equiv \frac{\lambda^2 D}{16} \mathcal{I}_{12} \tag{70}$$

with $\mathcal{I}_{12}$ given by 64 has dimension $[\Gamma_\phi] = \text{time}^{-1}$.

The fluctuation ratio,

$$F(\omega) = 2N_B(\omega)/\rho_B(\omega)$$

is also dimensionless and the collapse rate also has dimensions of time^{-1} as required.

B. Renormalization

The imaginary and real parts of the influence functional, 15, generate both dissipative and dispersive contributions, respectively. The dispersive contributions renormalize local operators in the effective 4D theory.

The mass is renormalized as follows:

$$\delta m^2 \propto \lambda \langle \mathcal{B} \rangle \sim \lambda \sum_{n\neq 0} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{n,\mathbf{k}}}$$

which is UV divergent and absorbed into the renormalized mass m_R^2.

The coupling is renormalized as well. Loop corrections from the KK modes renormalize λ and generate higher power operators ϕ^6 and ϕ^8 suppressed by the cutoff.

The spectral density includes an effective UV cutoff Ω (KK truncation or higher-dimensional completion). Physical observables are cutoff-independent once counterterms are included at the EFT matching scale.

The 4D zero-mode theory is, therefore, an effective field theory with renormalized parameters (m_R, λ_R) defined at a scale below Ω.

C. Lorentz Covariance

While the exact 5D theory is Lorentz covariant, the Markov approximation introduces a preferred time slicing through the replacement,

$$N(t - t') \to 2D\delta(t - t').$$

This approximation is defined in a rest frame of the stationary KK sector (or compactification background). Physically, the frame corresponds to the state in which the KK occupation $f_{n,\mathbf{k}}$ is stationary. Therefore, the reduced Markov dynamics are invariant only under spatial rotations in the bath rest frame. This is analogous to standard open-systems treatments and does not signal a fundamental violation of Lorentz symmetry.

D. Origin of the Stationary KK Occupation

Persistent decoherence requires a non-vacuum KK sector. Several physically motivated mechanisms can produce a stationary occupation:

1. Compactification-era particle production: rapid changes in the compact dimension generate KK excitations via nonadiabatic dynamics.

2. Hidden-sector entanglement: tracing over additional geometric or UV degrees of freedom leave the KK sector in a mixed state.

3. Many-mode dephasing: even a pure global state appears stationary when coarse-grained over a dense spectrum with incommensurate phases.

In all these cases, the requirement is not a thermal distribution satisfying a KMS state but stationarity and finite low-frequency noise power, given by the scale Θ_{KK}.

E. Energy Balance and Heating

Collapse-like dynamics can lead to heating, which is a significant bound on CSL and other objective collapse theories. The rate of change for the system obeying the master equation 69 is,

$$\frac{d}{dt}\langle H\rangle = -\Gamma_\phi \langle [\phi^2, [\phi^2, H]]\rangle.$$

Take a free scalar field,

$$H = \int d^3x \left[\frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m_R^2\phi^2 \right].$$

The double commutator generates higher-order field correlations but does not generically lead to runaway energy production in weak-coupling regimes.

A conservative bound from dimensional analysis is,

$$\frac{d}{dt}\langle H \rangle \sim \Gamma_\phi \langle \phi^4 \rangle.$$

This is required to be below experimental heating bounds, which constrain $\lambda^2 R \Theta_{\mathrm{KK}}$ to be sufficiently small. A full phenomenological analysis is beyond the scope of this work, but the scaling relations above already constrain the allowed parameter space. This is discussed further relative to experimental bounds in Section IV G.

F. Nonrelativistic Limit and CSL

To compare more directly to CSL, consider the nonrelativistic limit of the scalar field,

$$\phi(x) \sim \frac{1}{\sqrt{2m}}(\psi(x)e^{-imt} + \psi^\dagger(x)e^{imt})$$

where ψ is a Schrödinger field.

Then,

$$\phi^2 \sim \frac{1}{2m}\psi^\dagger\psi + \text{oscillatory terms.}$$

Averaging over fast oscillations, the Lindblad operator becomes proportional to the particle density,

$$L \propto \psi^\dagger\psi.$$

This means that the relativistic ϕ^2 theory reduces to mass-density decoherence in the nonrelativistic limit, aligning with the model of CSL.

G. Experimental Constraints and Distinguishing Predictions

The collapse rate 56 should be compared to CSL rates γ_{CSL} inferred from interferometry and spontaneous heating bounds. Matching these parameters defines the parameters region and the dependence of γ_{KK} provides a direct bridge between the microscopic theory and experiment.

Define a hidden KK sector noise strength of

$$\Xi_{\text{KK}} \equiv \frac{R\Theta_{\text{KK}}}{16}\mathcal{I}_{12}.$$

This quantity encapsulates the low-frequency noise strength of the hidden sector. The effective collapse rate is then

$$\gamma_{\text{KK}} = \frac{\lambda^2}{16}\Xi_{\text{KK}}.$$

This means that,

$$\Xi_{\text{KK}} = \frac{16\gamma_{\text{target}}}{\lambda^2}$$

where γ_{target} enables a more direct comparison to CSL because of how CSL is typically expressed with a collapse rate $\lambda_{\text{CSL}} \sim 10^{-16}\,\text{s}^{-1}$ and a correlation length scale $r_C \sim 10^{-7}\,\text{m}$ [18][19]. For a target rate of $10^{-16}\,s^{-1}$, for example, and a $\lambda = 0.1$, the noise product would be 1.6×10^{-15}, for example, and changes with collapse rate linearly and with λ according to the inverse square.

In the nonrelativistic limit $\phi^2 \to \frac{1}{2m}\psi^\dagger\psi$ such that the Lindblad operator becomes density-like, a physically finite model must smear density over a length d

$$L(\mathbf{x}) = \sqrt{\gamma_0}\int d^3y\, g_d(\mathbf{x}-\mathbf{y})\psi^\dagger(\mathbf{y})\psi(\mathbf{y})$$

with

$$g_d(\mathbf{x}) = \frac{1}{(2\pi d^2)^{3/2}}e^{-|\mathbf{x}|^2/2d^2}$$

The master equation for this is,

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{\gamma_0}{2}\int d^3x\,[L(\mathbf{x}),[L(\mathbf{x}),\rho]].$$

For a single nonrelativistic particle, the associated energy growth is,

$$\frac{dE}{dt} = \frac{3\hbar^2\gamma_0}{4md^2}. \tag{71}$$

This is the compact dimension model's analogous equation to CSL's standard heating effect, a major source of experimental bounds.

Any viable parameter choice must satisfy,

$$\gamma_0 < \frac{4md^2}{3\hbar^2}\left(\frac{dE}{dt}\right)_{\max}.$$

Particle	dE/dt per $\gamma_0 = 1\,\mathrm{s}^{-1}$
nucleon	$4.98 \times 10^{-28}\,\mathrm{J/s} \approx 3.11 \times 10^{-9}\,\mathrm{eV/s}$
electron	$9.16 \times 10^{-25}\,\mathrm{J/s} \approx 5.71 \times 10^{-6}\,\mathrm{eV/s}$

TABLE I. Single-particle heating rate for $d = 10^{-7}\,\mathrm{m}$ and $\gamma_0 = 1\,\mathrm{s}^{-1}$

Using the approximation,

$$\gamma_0 \sim \frac{\lambda^2}{256} R\Theta_{\mathrm{KK}},$$

this implies,

$$\lambda^2 R\Theta_{\mathrm{KK}} < \frac{1024 m d^2}{3\hbar^2}\left(\frac{dE}{dt}\right)_{\max}$$

which is an experimental constraint on the noise strength. The approximate particle heating for a nucleon and electron is given in Table I.

For a nucleon, given a heating allowance of $10^{-32}\,\mathrm{W}$ for example, the upper bound on γ_0 is $2.0 \times 10^{-5}\,\mathrm{s}^{-1}$. For an electron, the upper bound on γ_0 would be $1.1 \times 10^{-8}\,\mathrm{s}^{-1}$. These increase linearly with increasing allowance.

Heating bounds constrain the same low-frequency hidden-sector noise strength that controls collapse-like decoherence. For a smeared density Lindblad operator with smearing scale d, the single particle energy growth is 71. Thus a target CSL-like rate cannot be chosen independently of the allowed heating rate. The compact dimension model is viable only in the window where γ_{KK} reproduces the desired collapse phenomenology while γ_0 remains below the experimental heating limits.

This means that the model is viable when,

$$\gamma_{\mathrm{KK}} \sim \gamma_{\mathrm{CSL}}$$

while simultaneously satisfying heating constraints as controlled by the parameters $(\lambda, R, \Theta_{\mathrm{KK}}, d)$. This makes the model testable.

The model is also distinguishable from CSL by the following:

1. Finite-R recoherence corrections at short times.

2. Dependence on bandwidth used to probe (e.g., wavepacket bandwidth)

3. Deviations from perfect white noise at high frequencies

4. Potential collider signatures of KK modes.

V. CONCLUSION

It has been shown in this paper that for a small compact radius, dense KK spectrum, and stationary occupation modes, a unitary quantum field theory is able to reproduce collapse-like dynamics as in CSL. Deviations from CSL are suppressed by weak coupling and the tiny size of the compact dimension. Noise and dissipation kernels have been derived in an open, non-equilibrium Feynman-Vernon formalism, leading to an effective semi-classical Langevin equation in the zero-mode. Recoherence is also possible at very short time scales but suppressed by the dense modes and small radius, such that collapse becomes irreversible. As modes become dense, in a weak coupling regime, the nonlocal, non-Markovian dynamics become Markovian. The conclusion is that a modification of quantum mechanics may be unnecessary to achieve collapse-like behavior. Given stationary occupation modes derived from initial conditions, this collapse-like behavior can appear indistinguishable from CSL within certain experimental parameter bounds. The larger interpretation is that classical dynamics rests upon a foundation of hidden quantum entanglement information. This also indicates that the arrow of time, in this case, is predicted by a combination of factors, including the initial conditions within the compact dimension and the stationary occupation. Future work will extend this to vector and spinor fields and investigate initial conditions and the KMS condition for a thermal bath.

Appendix A: Keldysh Contour Theory

In an open quantum system such as this, the quantum theory splits the density matrix into a system and a bath such that [20][21],

$$\rho_{\text{sys}}[\phi, \phi'] = \int \Pi_{n\neq 0} D\chi_n \rho_{\text{tot}}.$$

In equilibrium (zero-temperature) quantum field theory, one begins with an initial state at $t = -\infty$ specified by a many-body density matrix $\hat{\rho}(-\infty)$. The density matrix evolves according to the Heisenberg picture,

$$\partial_t \hat{\rho}(t) = -i[\hat{H}(t), \hat{\rho}(t)].$$

where $\hbar = 1$.

This is solved by,

$$\hat{\rho}(t) = \hat{\mathcal{U}}_{t,-\infty} \hat{\rho}(-\infty) \hat{\mathcal{U}}_{-\infty,t}$$

where the evolution operator is given by the time-ordered exponent:

$$\mathcal{U}_{t,t'} = \mathbb{T} \exp \left(-i \int_{t'}^{t} d\tau \hat{H}(\tau) \right).$$

An observable at time t is given by,

$$\langle \hat{O}(t) \rangle \equiv \frac{\text{Tr}\hat{O}\hat{\rho}(t)}{\text{Tr}\hat{\rho}(t)} = \frac{1}{\text{Tr}\hat{\rho}(t)} \text{Tr}\hat{\mathcal{U}}_{-\infty,t} \hat{O} \hat{\mathcal{U}}_{t,-\infty} \rho(\hat{-}\infty), \tag{A1}$$

where the traces are done over the many-body Hilbert space.

The right-hand side of A1 says that one gets the expectation value by calculating the evolution from $t = -\infty$ to t and backwards to $t = -\infty$. This is avoided, normally, by a trick.

In equilibrium theory, one can assume that the state at $t = -\infty$ and the state at $t = \infty$ are the same, in which case one can remove the backwards evolution, but not so in the non-equilibrium case. Schwinger was the first to recognize that the evolution of nonequilibrium quantum field theory takes place along a closed time contour, forwards from $t = -\infty$ to time t and back again in a closed loop C_ν.

The basic premise of non-equilibrium quantum theory is to begin with a system evolving under a Hamiltonian,

$$H = h + H'(t)$$

where $h = H_0 + H_i$ is time independent with H_0 being the diagonal part and H_i being the non-diagonal (interacting) part and $H'(t)$ is time dependent. The equilibrium density matrix is,

$$\rho(h) = \frac{\exp(-\beta h)}{\mathrm{Tr}[\exp(-\beta h)]}$$

and the goal is to calculate the expectation of an observable O for times $t \geq t_0$ (where t_0 is often taken to be $-\infty$):

$$\langle O(t) \rangle = \mathrm{Tr}[\rho(h) O_H(t)]$$

which indicates that O_H is governed by the full Hamiltonian H and O is written in the Heisenberg picture.

Typically, this is attacked by perturbation theory with O_H being replaced with O_{H_0} and perturbations based on $H'(t)$ added as corrections.

In Keldysh's non-equilibrium formalism [4], instead of the time-ordering operator used in equilibrium theory, a contour ordering operator is used, which orders time-labels to their order on the Keldysh contour. The contour ordering Green's function is defined as (in Euclidean time),

$$G \equiv \langle \phi\phi \rangle \tag{A2}$$

which satisfies the same Dyson equation as in the equilibrium case.

The time ordering of Green's function in Keldysh formalism is as follows:

$$G(x - x') = \begin{cases} G^{++}(x - x') & t_1, t_1' \in C_1 \\ G^{>}(x - x') & t_1 \in C_2, t_1' \in C_1 \\ G^{<}(x - x') & t_1 \in C_1, t_1' \in C_2 \\ G^{--}(x - x') & t_1, t_1' \in C_2 \end{cases} \tag{A3}$$

where C_1 is the part of the contour on the path from $-\infty$ to ∞ and C_2 is the part from ∞ to $-\infty$. This means that C_1 is earlier in a contour sense than C_2.

This is handled by splitting the bosonic field into two parts (doubling degrees of freedom).

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